Introduction to Data Science

Analysis of LOCUS Scores

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Introduction

Background

- 1555 students took pilot course Introduction to Data Science (IDS)
- Student performance was measured before and after completing IDS, using Levels of Conceptual Understanding in Statistics (LOCUS), see `https://locus.statisticseducation.org/`
- Research Question: Which parameters influence student performance?

Difficulties

- Some students have missing pretest and/or posttest scores
- The two LOCUS forms A (pretest) and B (posttest) were not administered to all students as designated

$\Rightarrow$ Original statistical analysis did not take into account missing values

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Handling Missing Observations

Original Analysis

$y_j^{post} = \beta_0 + \beta_1 y_j^{pre} + \dots$

- can not handle missing pretest or posttest score

New Analysis

$y_j = \beta_0 + \beta_1 \, TestTime_j + \dots$

- missing values do not matter
- two observations for students with pretest (TestTime $= 0$) and posttest (TestTime $= 1$) score, one observation for students with pretest or posttest score, no observations for students without any scores

for student/observation j.

Interpretation of Additional Parameters

Original Analysis

$y_j^{post} = \beta_0 + \beta_1 y_j^{pre} + \beta_2 PLE_j + \dots$

- β_0: estimate for reference group
- β_2: effect of PLE on posttest adjusted for pretest

New Analysis

$y_j = \beta_0 + \beta_1 TestTime_j + \beta_2 PLE_j + \beta_3 \{PLE * TestTime\}_j + \dots$

- β_1: improvement from pretest ($\text{TestTime} = 0$) to posttest ($\text{TestTime} = 1$) in reference group
- β_2: effect of Primary Language English (PLE) on pretest
- β_3: effect of PLE on improvement from pretest to posttest

for student/observation j.

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Mixed Model Representation of Rasch Model

Rasch Model (Rasch, 1993)

$$\mathbb{P}(y_{ij} = 1) = \frac{1}{1 + \exp\big(-(b_j - \delta_i)\big)}$$

$$\Leftrightarrow \text{logit}\big(P(y_{ij} = 1)\big) = b_j - \delta_i$$

with

- b_j: the ability of student j
- δ_i: the difficulty of question i

Equivalent Mixed Model (Kamata, 1998, 2001)

$$\text{logit}\big(\mathbb{P}(y_{ij} = 1)\big) = b_j - \delta_i$$

with

- b_j: a random intercept for student j
- δ_i: a fixed effect for question i

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Proposed Model Formula

$$\begin{aligned}\text{logit}\big(\mathbb{P}(y_{ijk} = 1 | x_{ijk})\big) &= \beta_0 + \delta_i + b_j + d_k + h_k \mathit{TestTime} + \\ &\quad \beta_1 \mathit{TestTime} + \beta_2 \mathit{PLE} + \beta_3 \{\mathit{PLE} * \mathit{TestTime}\} + .\end{aligned}$$

with

- $-\delta_i$: fixed effect for question i ($=$ question difficulty)
- b_j: random intercept for student j ($=$ student ability)
- d_k: random intercept for teacher k on pretest
- h_k: random slope for TestTime per teacher k (teacher effect on improvement from pretest to posttest)
- β_1: improvement in reference group
- β_2: effect of PLE (pretest)
- β_3: effect of PLE on improvement

for question i, student j, and teacher k.

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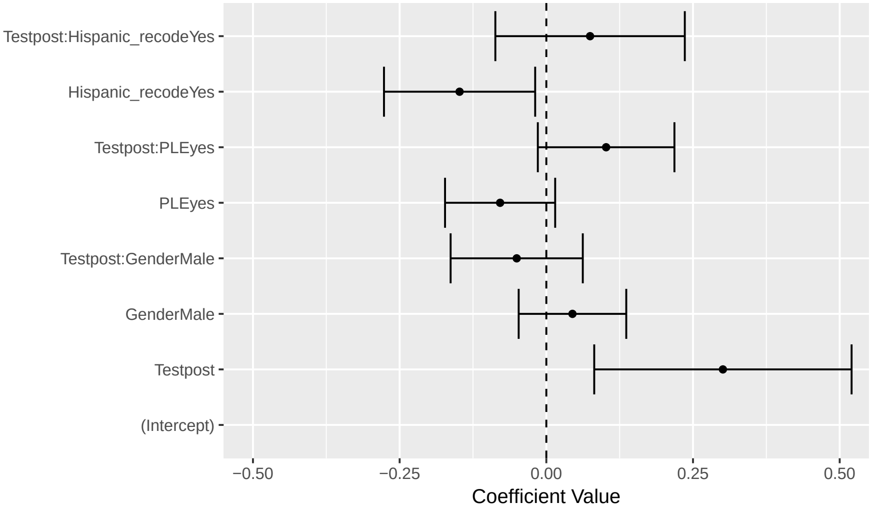
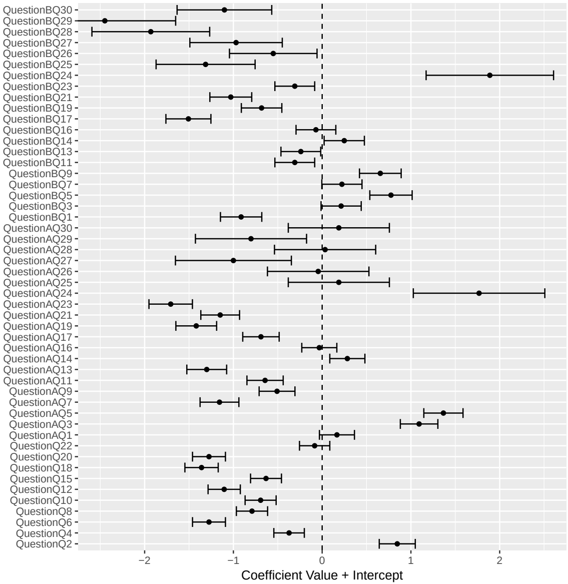
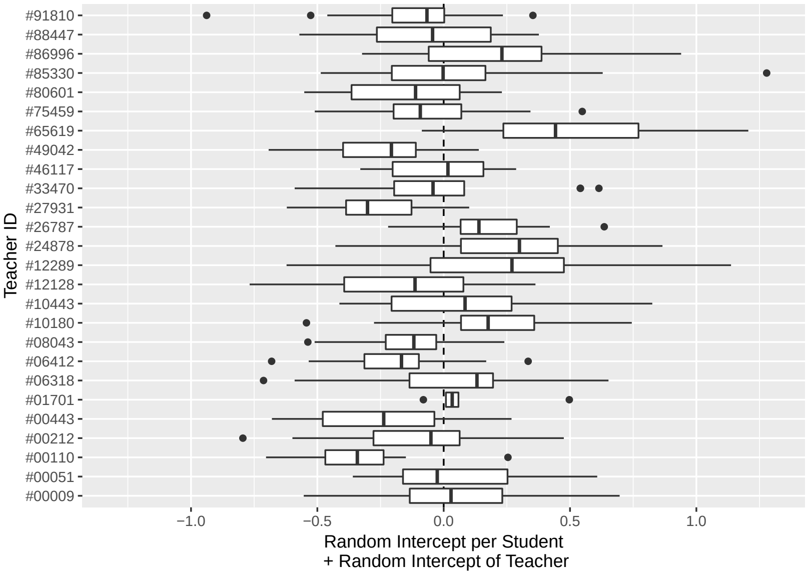
Coefficient Plot: Fixed Effects



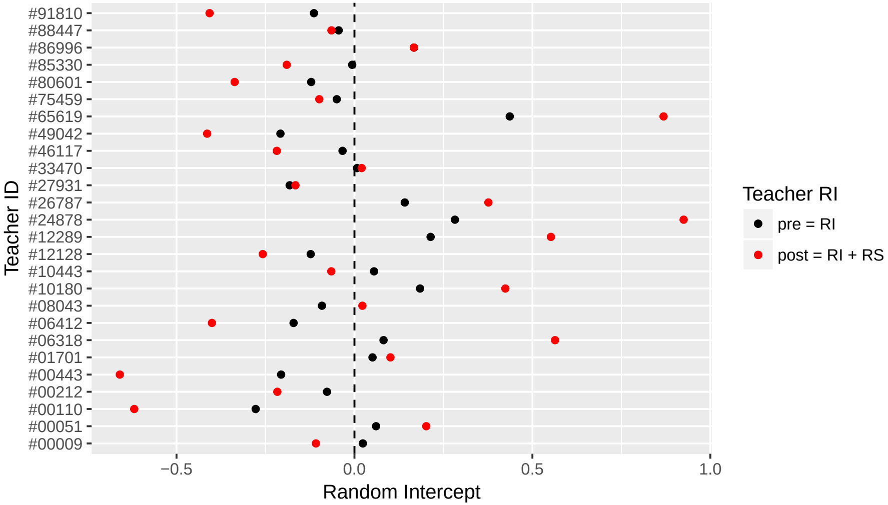
Coefficient Plot: Question Difficulty



Coefficient Plot: Student Ability



Coefficient Plot: Teacher Effects



Literature

Kamata, A. (1998). One-parameter hierarchical generalized linear logistic model: An application of HGLM to IRT.

Kamata, A. (2001). Item analysis by the hierarchical generalized linear model. *Journal of Educational Measurement 38*(1), 79–93.

Rasch, G. (1993). *Probabilistic models for some intelligence and attainment tests.* ERIC.

Additional Material

R Code for Mixed Model

```
mgcv::gamm(Score ~ Question +
             Test * (Gender + Hispanic + PLE),
           random = list(Teacher_ID = ~1 + Test,
                         LAUSD_ID = ~1),
           family = "binomial",
           data = student_data_long)
```

Original Analysis

First Step: Rasch Model

$$\text{logit}\big(P(y_{ijk} = 1)\big) = b_{jk} - \delta_i$$

with

- b_{jk}: the ability of student j for teacher k
- δ_i: the difficulty of question i

Second Step: Mixed Model

$$b_{jk} = \beta_0 + h_k + \beta_1 y_{jk}^{pre} + \beta_2 PLE + \dots$$

with

- b_{jk}: estimated student ability from Rasch model
- h_k: random intercept for teacher k
- β_1 effect of pretest score on student ability
- β_2 effect of PLE on student ability